Paper II

Total Marks: 100

Ph.D. Coursework Examination Syllabus (Mathematics)

Sr. No.	Entrance Exam Course Content
1	**Probability Concepts:** Introduction to Probability, Theorems of Probability. **Discrete Probability Distributions:** The Binomial Distribution, The Poisson Distribution, The Geometric Distribution, The Hypergeometric Distribution. **Continuous Probability Distributions:** The Normal Distribution, The Exponential Distribution, Mathematical Expectation.
2	**Calculus:** **Differential Calculus:** Functions of One Variable, Limit, Continuity and Differentiability, Mean Value Theorems, Applications, Partial Differentiation. **Integral Calculus:** Indefinite Integrals, Definite Integrals, Applications, First Order Differential Equations, Existence and Uniqueness of Solutions of Initial Value Problems (IVPs) for First Order Ordinary Differential Equations, Singular Solutions of First Order ODEs, System of First Order ODEs.
3	**Linear Algebra:** Algebra of Matrices and Determinants, System of Linear Equations, Vector spaces, Subspaces, Linear Dependence, Basis, Dimension, Algebra of Linear Transformations. Rank and Determinant of Matrices. Eigenvalues and Eigenvectors, Cayley-Hamilton Theorem. Matrix Representation of Linear Transformations. Change of Basis, Canonical Forms, Diagonal Forms, Triangular Forms, Jordan Forms. Inner Product Spaces, Orthonormal Basis. Quadratic Forms, Reduction and Classification of Quadratic Forms.
4	**Differential Equations:** **Ordinary Differential Equations:** Second Order Ordinary Differential Equations, Higher Order ODEs, Wronskian, General Theory of Homogenous and Non-homogeneous Linear ODEs, Method of Variation of Parameters, Sturm-Liouville Boundary Value Problem, Green's Function, Applications. **Partial Differential Equations:** Solution by Separating Variables, Solution of PDEs by Laplace Transforms, Lagrange and Charpit Methods for Solving First Order PDEs, Cauchy Problem for First Order PDEs. Classification of Second Order PDEs, General Solution of Higher Order PDEs with Constant Coefficients, Method of Separation of Variables for Laplace, Heat and Wave equations.
5	**Analysis:**

Elementary Set Theory, Finite, Countable and Uncountable Sets, Real Number System as a Complete Ordered Field, Archimedean Property, Supremum, Infimum. Sequences and Series, Convergence, Limsup, Liminf. Bolzano Weierstrass Theorem, Heine Borel Theorem. Continuity, Uniform Continuity, Differentiability, Mean Value Theorem. Sequences and Series of Functions, Uniform Convergence. Riemann Sums and Riemann Integral, Improper Integrals. Monotonic Functions, Types of Discontinuity, Functions of Bounded Variation, Lebesgue Measure, Lebesgue Integral. Functions of Several Variables, Directional Derivative, Partial Derivative, Derivative as a Linear Transformation. Metric Spaces, Compactness, Connectedness. Normed Linear Spaces. Spaces of Continuous Functions as Examples.

6	**Algebra:**
	Permutations, Combinations, Pigeon-hole Principle, Inclusion-Exclusion Principle, Derangements. Fundamental Theorem of Arithmetic, Divisibility in Z, Congruences, Chinese Remainder Theorem, Euler's Ø- Function, Primitive Roots. Monoids, Groups, Subgroups, Normal Subgroups, Quotient Groups, Homomorphisms, Cyclic Groups, Permutation Groups, Cayley's Theorem, Class Equations, Sylow Theorems. Rings, Ideals, Prime and Maximal Ideals, Quotient Rings, Unique Factorization Domain, Principal Ideal Domain, Euclidean Domain. Polynomial Rings, and Irreducibility Criteria. Fields, Finite Fields, Field Extensions.
7	**Numerical Analysis:**
	Numerical Solutions of Algebraic Equations, Method of Iteration and Newton-Raphson Method, Rate of Convergence, Solution of Systems of Linear Algebraic Equations Using Gauss Elimination and Gauss-Seidel Methods. Finite Differences, Lagrange, Hermite and Spline Interpolation, Numerical Differentiation and Integration, Numerical Solutions of ODEs Using Picard, Euler, Modified Euler and Runge-Kutta Methods.
8	**Complex Analysis:**
	Algebra of Complex Numbers, The Complex Plane, Polynomials, Power Series, Transcendental Functions such as Exponential, Trigonometric and Hyperbolic Functions. Analytic Functions, Cauchy-Riemann Equations. Contour Integral, Cauchy's Theorem, Cauchy's Integral Formula, Liouville's Theorem, Maximum Modulus Principle, Schwarz Lemma, Open Mapping Theorem. Taylor Series, Laurent Series, Calculus of Residues. Conformal Mappings, Mobius Transformations (Bilinear Transformations).
9	**Descriptive Statistics:**
	Independent Random Variables, Marginal and Conditional Distributions. Characteristic Functions. Probability Inequalities (Tchebyshef, Markov, Jensen). Modes of Convergence, Weak and Strong Laws of Large Numbers, Central Limit Theorems (i.i.d. case).
	Markov Chains with Finite and Countable State Space, Classification of States, Limiting Behaviour of n-step Transition Probabilities, Stationary Distribution. Standard Discrete and Continuous Univariate Distributions. Sampling Distributions. Standard Errors and Asymptotic Distributions, Distribution of Order Statistics and Range.
	Methods of Estimation. Properties of Estimators. Confidence Intervals. Tests of Hypotheses: Most Powerful and Uniformly Most Powerful tests, Likelihood Ratio Tests. Analysis of Discrete Data and Chi-square Test of Goodness of Fit. Large Sample Tests. Simple Nonparametric Tests for One and Two Sample Problems, Rank Correlation and Test for Independence. Elementary Bayesian Inference.
	Gauss-Markov Models, Estimability of Parameters, Best Linear Unbiased Estimators, Tests for Linear Hypotheses and Confidence Intervals. Analysis of Variance and Covariance. Fixed, Random and Mixed Effects Models. Simple and Multiple Linear Regression. Elementary Regression Diagnostics. Logistic Regression.
	Multivariate Normal Distribution, Wishart Distribution and Their Properties. Distribution of Quadratic Forms. Inference for Parameters, Partial and Multiple Correlation Coefficients and Related Tests. Data Reduction Techniques: Principle Component Analysis, Discriminant Analysis, Cluster Analysis, Canonical

Correlation. Simple Random Sampling, Stratified Sampling and Systematic Sampling. Probability Proportional to Size Sampling. Ratio and Regression Methods. Completely Randomized, Randomized Blocks and Latin-square Designs. Connected, Complete and Orthogonal Block Designs, BIBD. 2K Factorial Experiments: Confounding and Construction.

Series and Parallel Systems, Hazard Function and Failure Rates, Censoring and Life Testing.

Linear Programming Problem. Simplex Methods, Duality. Elementary Queuing and Inventory Models. Steady-state Solutions of Markovian Queuing Models: M/M/1, M/M/1 with Limited Waiting Space, M/M/C, M/M/C with Limited Waiting Space, M/G/1.